STUDY OF ABSORBING BOUNDARY CONDITIONS IN THE 3D-TLM SYMMETRICAL CONDENSED NODE MODEL

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ABSTRACT

The TLM method is a numerical technique based on temporal and spatial sampling of electromagnetic fields. As with the FD-TD method, the absorbing boundary conditions are needed to truncate computational regions when open structures are simulated. The Finite-Difference Time-Domain absorbing boundary conditions have been adapted to and implemented in the 3D-TLM symmetrical condensed node model. It is also demonstrated that instability may occur due to spurious modes of the 3D-TLM condensed node mesh.

1 Introduction

The Transmission Line Matrix method (TLM) has been extensively applied to solve electromagnetic wave propagation, diffusion and network problems in the time-domain[1]. Given the flexibility and simplicity of its basic algorithm, TLM can handle arbitrary geometries and account for realistic features that are often neglected with other methods.

Just like the FD-TD scheme, the TLM method requires absorbing boundary conditions to truncate the computational domain in order to characterize open structures. Several schemes have been proposed for the two-dimensional TLM model, and good results were obtained [2]-[4]. We have implemented one of these approaches, the absorbing boundary condition developed by Higdon, into the 2D-TLM models and achieved wide-angle absorption. More recently, a quite simple formulation which can be derived by using Taylor's expansion formulae, was proposed by Saguet[5]. Both approaches work quite well for the 2D-TLM model and the 3D-TLM asymmetrical node.

2 Absorbing boundaries

Consider a discretized spatial domain $\Omega(x, y, z), x < 0$ where the absorbing boundary is placed at $x = 0$ as shown in Fig. 1. The absorbing boundary conditions must be constructed such that waves incident on the boundaries are absorbed rather than reflected. One of the commonly used approaches is to use extrapolation techniques, where the field values at absorbing boundaries are predicted, at each time step, by a function of field values at interior nodes ($x < 0$). In the TLM method, the field values can be considered proportional to impulses reflected by the boundaries and the inner nodes[6] .

For simplicity, we denote: $V(i_x\Delta x, i_y\Delta y, i_z\Delta z, n\Delta t) = V_{i_x}^n$, $Z^l V_{i_x}^n = V_{i_x-l}^n$, $K^l V_{i_x}^n = V_{i_x}^{n-l}$, where l, i_x, i_y, and i_z are integer numbers, Δx, Δy, Δz and Δt are the space and time increments and Z and K are the space and time operators, respectively.

By using the Taylor expansion of $V_{i_x}^n$, one has[5] for the first order approximation:

$$(1 - 2\, Z^{-1}\, K^{-1} + Z^{-2}\, K^{-2}) V_0^n = 0 \qquad (1)$$

and for the third order:

$$(1-2.5\,Z^{-1}K^{-1}+2\,Z^{-2}K^{-2}+0.5\,Z^{-3}K^{-3})V_0^n=0 \quad (2)$$

For Higdon's conditions, one can have the Pth order condition[7]:

$$\prod_{j=1}^{P}\left(\frac{1-Z^{-1}}{\Delta x}\frac{1+K^{-1}}{2}+\frac{cos(\theta_j)}{c}\frac{1-K^{-1}}{\Delta t}\frac{1+Z^{-1}}{2}\right)V_0^n=0 \quad (3)$$

where θ_j is the incident angle for which the waves can be exactly absorbed and c is the speed of light in the virtual medium being modelled.

The above absorbing boundary conditions have been applied to the two-dimensional TLM model and the three-dimensional asymmetrical TLM node model and good results have been obtained[4][5].

For the 3D-TLM symmetrical condensed node, (1) to (4) are still valid except that K^{-l} must be replaced by K^{-2l}. The replacement or modification is based on the fact that in the 3D-TLM symmetrical condensed node model, the scattered impulse on the link line of a node, due to an incident impulse with the same polarization on the link line at the other side of the node , only appears after two time steps, ie. $2\,\Delta t$.

The reflection coefficients of the above absorbing boundaries are shown in Fig. 2.

3 Spurious Modes And Their Effects At Absorbing Boundaries

In his paper, Nielsen has shown that due to periodic spatial sampling, there exist spurious modes in the 3D-TLM symmetrical condensed node model[8]. These spurious modes are nonphysical and possess high spatial variations. They can even propagate at low frequencies. When the absorbing boundary conditions are applied, these spurious modes can be amplified and then reflected back into the computational domain. Fig.3 shows the reflection coefficients for spurious modes. One can see that the reflection coefficients are larger than one. This means that spurious modes are indeed amplified by the absorbing boundary conditions. Thus, the reflected spurious modes, which are injected back into the computational domain, contain more energy than the incident spurious modes, which leads to instability.

Consider that any numerical wave equation solution can be expressed as a superposition of waves as follows:

$$V(i_x\Delta x,i_y\Delta y,i_z\Delta z,n\Delta t)=e^{j\omega n\Delta t-jk_xi_x\Delta x-jk_yi_y\Delta y-jk_zi_z\Delta z} \quad (4)$$

where k_x, k_y, k_z are the component propagation constants in X, Y and Z directions, respectively. They satisfy the dispersion relation as indicated in [8].

Therefore, in order to eliminate the spurious modes near absorbing boundaries, the following conditions must be satisfied:

$$k_x\Delta x\le\frac{\pi}{2} \quad (5)$$
$$k_y\Delta y\le\frac{\pi}{2} \quad (6)$$
$$k_z\Delta z\le\frac{\pi}{2} \quad (7)$$

A simulation of a section of rectangular waveguide terminated with absorbing boundaries at both ends was performed. If lower order modes are excited, for which (5) to (7) are satisfied, the absorbing boundaries work very well. Fig. 4 shows the VSWR in the waveguide. However, if higher order modes are excited, for which (5) to (6) are not satisfied, an unstable solution will be generated by the absorbing boundaries due to existence of spurious modes.

4 Conclusion

Generally, spurious modes are inherent in any numerical technique based on temporal and spatial sampling. If these nonphysical modes reach a boundary which is conceived for physical mode absorption, the boundary may reflect the spurious modes with amplification instead of attenuation, causing instability. This happens when some finite-difference time-domain absorbing boundary conditions are applied to the 3D-TLM symmetrical condensed node model. In conclusion, a stable absorbing boundary must be constructed in such a way that it absorbs both physical modes and spurious modes. Eswarappa et al. have proposed a dissipation techniques [2] by which both physical modes and spurious modes are gradually attenuated and absorbed at boundaries. A more efficient absorbing boundary condition for 3D-TLM condensed node scheme, including spurious modes, is currently under investigation.

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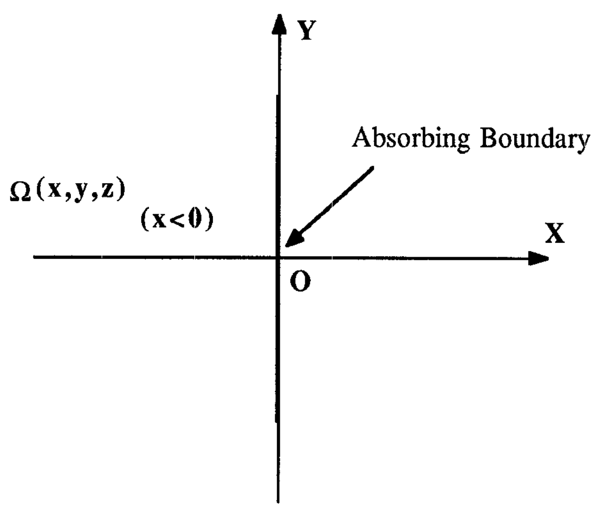

Fig.1 An Absorbing Boundary at x=0

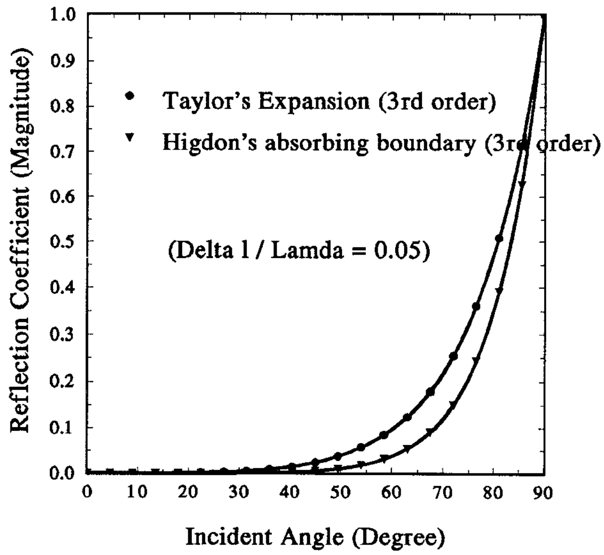

Fig.2 The Reflection Coefficients of the Absorbing Boundaries
(For the Physical Modes)

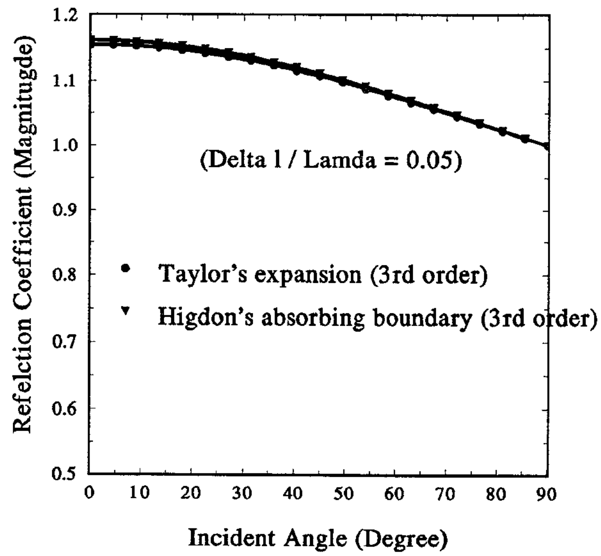

Fig.3 The Reflection Coefficients of the Absorbing Boundaries
(For the Spurious Modes)

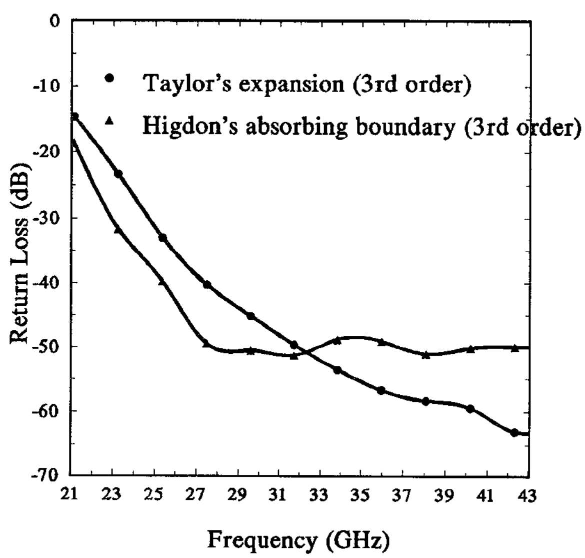

Fig.4 Return Loss of Two Back to Back Absorbing Boundaries In a Section of WR28 Rectangular Waveguide